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Theoretical Investigation of reflection from a one dimensional photonic crystal

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Abstract

In recent years, optical technology and photonics industry developed fast, but further progress became difficult due to a fundamental limit of light known as the diffraction limit. This limit could be overcome using the novel technology of nano-optics or nano-photonics in which the size of the electromagnetic field is decreased down to the nano-scale and is used as a carrier for signal transmission processing, and fabrication.

Among this systems we focus our interest of study a design for one dimensional photonic crystal (1D PC). we adopted the transfer matrix method and Bloch theorem are used to calculate the reflection coefficient and reflectivity of (1D-PC) periodic structure for TE and TM-modes at different angles of incidence. Results obtained showing the effect of the filling factor as well as the incident angle on the photonic band gap width. The analysis is carried out using MATLAB software tool.

Keywords: Photonic Band Gap, Transfer Matrix Method, Bloch Theorem, reflection.

Introduction

For the past of fifty years, semiconductor physics has played a vital role in almost every aspect of modern technology [1]. New research suggests that we may now be able to design structures to control the propagation of electromagnetic waves in the same way as the periodic potential in semiconductor crystals [2-13]. A new class of materials called photonic crystals and the stop-band is called a photonic band-gap (PBG) [3]. Photonic crystals (PhCs) are the novel class of optical media represented by natural or artificial structures with periodic modulation of the refractive index [4] .1D photonic crystals are the simplest structure in photonic crystal family [5]. 1D photonic crystals still possess many exciting properties such as adjustable dispersion and birefringence, acting as homogeneous materials [6]. Compared with 2D or 3D photonic crystals , the simple structure of 1D photonic crystals makes them easy to be integrated with the existing photonic devices without changing fabrication procedures [7-14] . 1D photonic crystals have been known for several decades as Bragg mirror, which work on the concept of standing wave phenomena due to the occurrence of constructive interference of the incident and reflected light waves [8]. The DBR has a multi-pair layered structure. Each pair is composed of two layers of different materials, which have different refractive indices [8-9]. The thickness of both the high and the low refractive index materials is a quarter of the designed wavelength [10]. The difference between the refractive indices and the number of pair in the DBR must be increased to achieve a higher reflectance [11]. The numerical modeling of photonics crystals is based on the calculation of the transmission and reflection coefficients properties [12]. These methods including the plane wave expansion (PWE) method, the finite difference time-domain (FDTD) method, and the transfer matrix method (TMM).

The transfer matrix method (TMM) is a very useful and simple mathematical method for the simulation of multilayers [13-15]. In this paper we study the reflectivity spectrum of multi-layered dielectric films of (1D- PC) structures composed of TiO_2/SiO_2 the dispersion in the wavelength range 600 - 1400 nm The center of the stop band was designed to be at 1000 nm a high reflectance of 91% and 99% was achieved using structures of 15 and 20 Tio_2/Sio_2 pairs, respectively. The reflectivity spectrum was simulated using the matrix method in optics.

Theory

For simplicity, we consider here progressive and plane waves crossing multilayers is normal incidence. The layers are considered as linear, homogeneous and perfect dielectrics. Non-absorbent and without charges neither currents. The layers are perpendicular to the z-axis and the ℓth layer has ε_ℓ as a dielectric constant and $\mu_\ell = \mu_0$ (non-magnetic material).

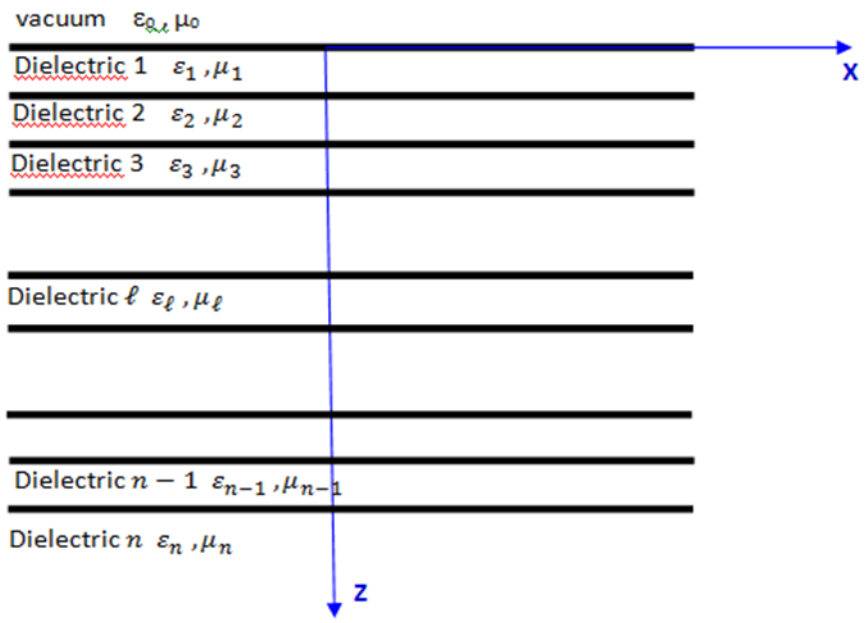

Figure 1: scheme of the multilayers submitted to normal incident electromagnetic waves.

The electromagnetic waves considered here, are oscillating with an angular frequency ω, traveling along the z- axis and *polarized on the x*-axis:

$$\vec{E} = E_x(z)e^{-i\omega t}\,\hat{x} \qquad (1)$$

$$\vec{H} = H_y(z)e^{-i\omega t}\,\hat{y} \qquad (2)$$

As both the E and H fields are continuous at boundary 1, one may write E_1 and H_1 as :

$$H_y(z=0) = -i\,\eta_\ell \sin(k_\ell z)\,E_x(z) + \cos(k_\ell z)H_y(z) \qquad (3)$$

$$E_x(z=0) = \cos(k_\ell z)\,E_x(z) + (-i\sin(k_\ell z)\frac{1}{\eta_\ell}H_y(z) \qquad (4)$$

were $\eta_l = \frac{k_l}{\omega\mu_\circ} = \frac{n_l}{\mu_\circ c}$ (5)

The refractive index of the layer l:

$$n_l = \sqrt{\frac{\varepsilon_l}{\varepsilon_\circ}} \qquad (6)$$

The wave number of the incident

$$k = \omega/c \qquad (7)$$

The velocity of light in the vacuum

$$c = \sqrt{\frac{1}{\mu_\circ \varepsilon_\circ}} \qquad (8)$$

the tangential components of E and H are continuous across a boundary, and since there is only a positive going wave in the substrate, a 2×2 matrix can be used to express the relationship which connects the tangential components of E and H at the incident interface with the tangential components of E and H which are transmitted through the final interface. This matrix is known as the characteristic matrix of the thin film.

$$\begin{bmatrix} E_x(z=0) \\ H_y(z=0) \end{bmatrix} = \begin{bmatrix} \cos(k_\ell z) & -\frac{i}{\eta_\ell}\sin(k_\ell z) \\ -\frac{i}{\eta_\ell}\sin(k_\ell z) & \cos(k_\ell z) \end{bmatrix} \begin{bmatrix} E_x \\ H_y \end{bmatrix} = M_\ell(z) \begin{bmatrix} E_x(z) \\ H_y(z) \end{bmatrix} \qquad (9)$$

$M_\ell(z)$ is a matrix describing the layer "ℓ":

$$\begin{cases} \eta_\ell = n_1 cos\theta_l \,, & TM\ mode \\ \dfrac{n_l}{\cos\theta_l} & TE\ mode \end{cases} \qquad (10)$$

$$\theta_l = sin^{-1}(sin\theta_i \frac{n_I}{n_i}). \qquad (11)$$

We can write this relation more generally, in the same medium and different values of z, as follow:

$$\begin{bmatrix} \mathrm{E_x(z)} \\ \mathrm{H_y(z)} \end{bmatrix} = M_\ell(\mathrm{z-z'}) \begin{bmatrix} \mathrm{E_x(z')} \\ \mathrm{H_y(z')} \end{bmatrix} \qquad (12)$$

Considering the structure of N period of layers with alternately refractive indices n_1 and n_2 and thickness d_1 and d_2, the coefficient of propagating states in right and left side of the multilayer structures are calculated by multiplying transfer matrices of each cell.

$$\begin{bmatrix} E_x(z=0) \\ H_y(z=0) \end{bmatrix} = M_1(z=0)M_2(z_2-z_1)M_3(z_3-z_2)\ldots\ldots M_N(z_N-z_{N-1}) \begin{bmatrix} E_x(z_N) \\ H_y(z_N) \end{bmatrix}$$

$$= M_1(h_1)M_2(h_2)\ldots\ldots M_N(h_N) \begin{bmatrix} E_x(z_N=h) \\ H_y(z_N=h) \end{bmatrix} \qquad (13)$$

Were $z_1 = h_1$, $z_2 - z_1 = h_2$, $z_3 - z_2 = h_3$, ... $z_l - z_{l-1} = h_l$ ... $z_N - z_{N-1} = h_N$ and $E_x(h)$ represent the transmitted electric field crossing the multilayers (see figure 1) .

Defining the reflection coefficient as $r = \frac{E_O}{E_r}$ and the transmission coefficient by $t = \frac{E_O}{E_t}$ and looking for their expressions taking into account the presence of the multilayers. Where $M = M_1(h_1)\ M_2(h_2)M_3(h_1) \dots \dots .. M_N(h_N)$.

This give us, after simple algebra that the expressions of the coefficient of transmission :

$$t = \frac{2}{M_{11}+\eta_f M_{12}+\frac{M_{21}}{\eta_0}+\frac{\eta_f}{\eta_0}M_{22}} \tag{14}$$

and the coefficient of reflection as:

$$r = \frac{M_{11}+M_{12}\eta_f-\frac{M_{21}}{\eta_0}+M_{22}\frac{\eta_f}{\eta_0}}{M_{11}+M_{12}\eta_f+\frac{M_{21}}{\eta_0}+M_{22}\frac{\eta_f}{\eta_0}} \tag{15}$$

High-reflectance coatings

For a high-reflectance film with N pairs of high- and low-refractive index layers

$$M = (M_{H1}M_{L1})(M_{H2}M_{L2}) \dots (M_{HN}M_{LN}) = (M_H M_L)^N = M_{HL}^N$$

$$M = \begin{bmatrix} \left(-\frac{n_2}{n_1}\right)^N & 0 \\ 0 & \left(-\frac{n_1}{n_2}\right)^N \end{bmatrix} \begin{bmatrix} \left(-\frac{n_2}{n_1}\right)^N & 0 \\ 0 & \left(-\frac{n_1}{n_2}\right)^N \end{bmatrix} \tag{16}$$

The power reflectance coefficients for the multi-layer film at normal incidence are given by :

$$R = \frac{\left(n_0\left(\left(-\frac{n_2}{n_1}\right)\right)^N - n_f\left(\left(-\frac{n_1}{n_2}\right)\right)^N\right)^2}{\left(n_0\left(\left(-\frac{n_2}{n_1}\right)\right)^N + n_f\left(\left(-\frac{n_1}{n_2}\right)\right)^N\right)^2} \qquad (17)$$

$$R = \frac{\left[1 - \frac{n_f}{n_0}\left(\left(\frac{n_1}{n_2}\right)\right)^{2N}\right]^2}{\left[1 + \frac{n_f}{n_0}\left(\left(\frac{n_1}{n_2}\right)\right)^{2N}\right]^2} \qquad (18)$$

Numerical Calculation and Discussion

Number of periods of the multilayer

The effect of the number of periods on reflectivity for a variable number of periods with low and high index contrast. The refractive index for Tio_2Sio_2 is n_h =2,4, $n_l = 1.45$, $n_s = 1.45$. The wavelength range is $600 - 1400\ nm$. The stop band was found to be centered at $= 1000$ in the range $600 - 1400\ nm$ in the infrared region. Shown in Fig. 2 The results of reflectivity against wavelength with a varied number of periods given by the TMM simulate.

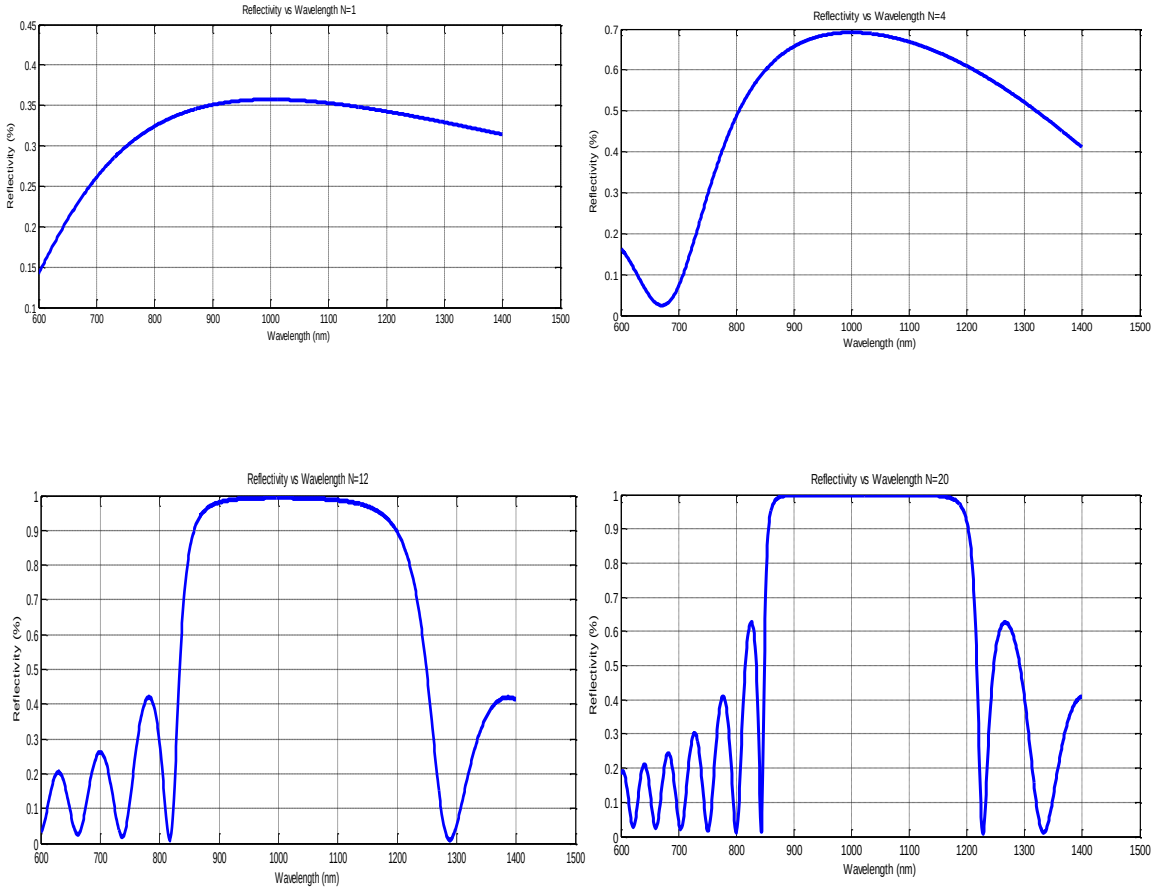

Fig.2 reflectivity spectra of a Tio_2Sio_2 with $n_h = 2.4 n_l = 1.45$ and number of Periods 1,4,12 and 20.

From figure 2, we observe that when the number of periods N=1 the reflectivity R=0.35 , N=4, the reflectivity R=0.7, N=12, the reflectivity R=0.97 and N=20, the reflectivity R=1. We conclude The number of periods (N) influences on different characteristics of the Band gap. The increase in the number of periods N leads to an increase of the reflectivity within the band gap, and enlarges its width. The band edges also become sharper.

Varying Angle of Incidence

The reflectance of a high reflection structure consisting of 24 layers of a Tio_2Sio_2 . Applying transfer matrix method, we plotted reflectivity of the structures with wavelength for various angles of incidence see fig.3,4 . We observe when the incident angle increase, the admittance of TE polarization increases and that of TM polarization decreases. So the reflection bandwidth of TE polarization is wider than that at normal incidence, and that of TM polarization is narrower but the center wavelength will shift towards shorter wavelength. Therefore, at a high incident angle, the reflectance of TM-polarized light may be quite low in contrast to the high reflectance of TE-polarized light. As we see in Table 1 move to the shorter wavelengths of TE wave as compared with TM wave for different angles of incidence.

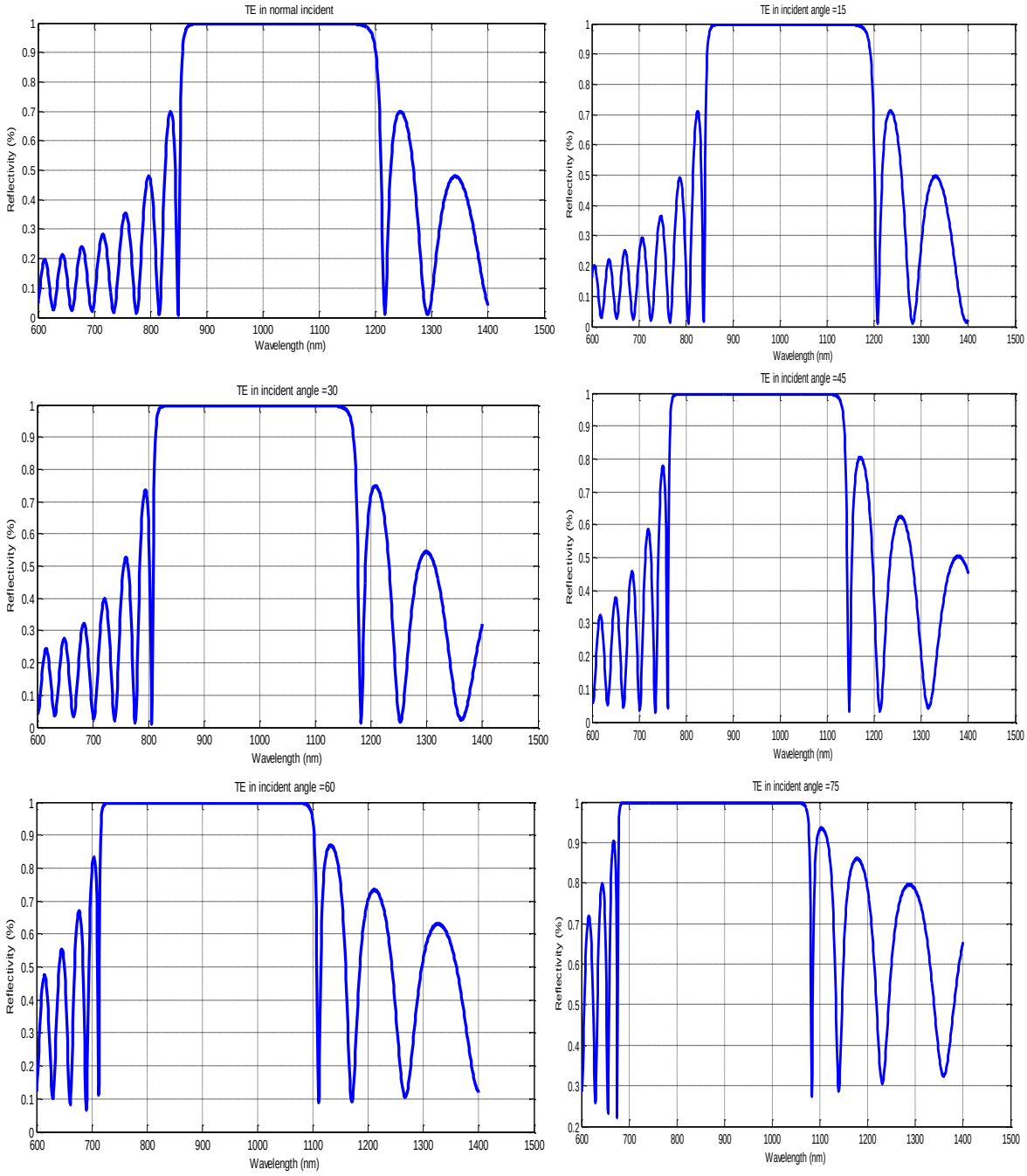

Fig. 3 Reflectivity curve Vs. wavelength (nm) of Tio_2Sio_2 multilayer structure for TE mode at different angle.

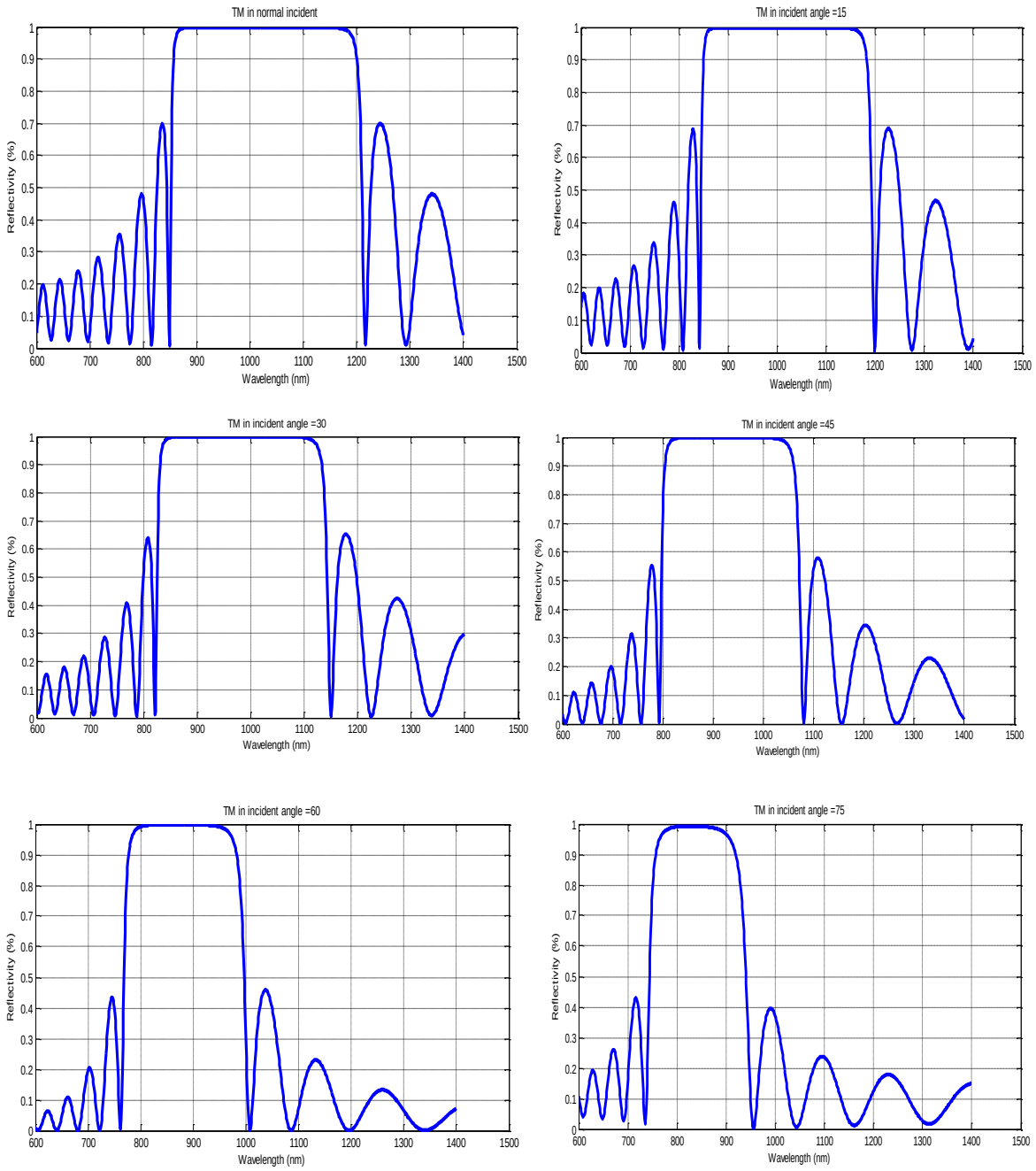

Fig. 4 Reflectivity curve vs. wavelength (nm) of Tio_2/Sio_2 multilayer structure for TM mode at different angles.

Angle of incidence, θ (deg.)	TE (nm)	Band width (nm)	TM (nm)	Band width (nm)
0	850-1200	350	855-1210	355
15	840-1200	360	851-1200	349
30	800-1180	380	830-1150	320
45	770-1152	382	798-1090	292
60	710-1100	390	775-1000	225
75	675-1086	411	745 – 950	205

Table 1. The bandwidth of TE waves as compared with TM wave for different angles of incidence.

From the plots of reflection spectra of TiO_2 / SiO_2 for different angles of incidence. It is clear from Table 1 that the reflectance range for TE mode is 850-1200 nm at normal incidence. Similarly, for angles of incidence of 15°, 30°, 45°, 60°, 75° the reflectance ranges are 840-1200 nm, 800-1180 nm; 770-1152 nm; 710-1100 nm, 675-1086 nm for TE mode. Also, the ranges of reflectance for TM mode are 855-1210 nm, 851-1200 nm; 830-1150 nm; 798-1090, 775-1000, 745-950 nm; for angles of incidence at 0°, 15°, 30°, 45°, 60°, 75° respectively.

Conclusion

In summary, we have used the transfer matrix method and Bloch theorem to study the calculation of reflectivity and dispersion relation for both TE and TM modes at different incidence angles. From above results can be concluding that the simulation results also produce highly correlated results with the theoretical prediction. Our results show that the increased of reflectivity depend on many factors, such as the number of periods. The angle of incidence of the wave is also another factor which affects the width of band gaps. Such a structure can be used as a very useful optical device in the optical industry.

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